XXXV. *A Memoir concerning the most advantageous Construction of Water-Wheels, etc. by Mr.* Mallet *of* Geneva. *Communicated by* M. Maty, *M. D. Sec. R. S. Translated from the* French, *by* J. Bevis, *M. D. R. S. S. Read* March 26, 1767.

§I $\mathbf{T}$HE stream of rivers is of such importance in moving machines of all kinds, that any attempt towards perfecting this part of mechanics may be considered as of very great utility.

The first wheel on which the stream acts is one of the most essential members of the machine, and it is easy to discern that the greater or less effect thereof must depend, in a great measure, on the manner of constructing this wheel, and on the dimensions given to it. I shall not at present inquire whether wheels of different constructions from those which have been long in use might be advantageously substituted in their stead; but confine myself, in this essay, to an examination of the most common ones, and to discover the means by which they may be made to produce the greatest possible effects. Their construction is very simple; they consist of several planes inserted into the same axle placed horizontally above the surface of the water, and in a position perpendicular to the stream. These planes, called
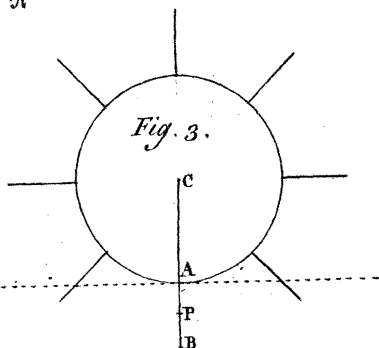
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Fig. 1.
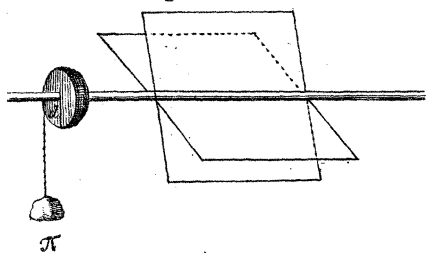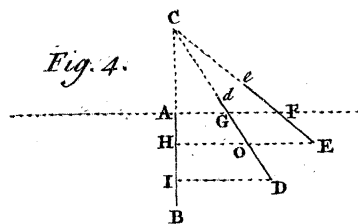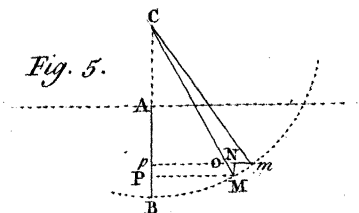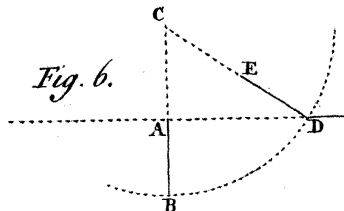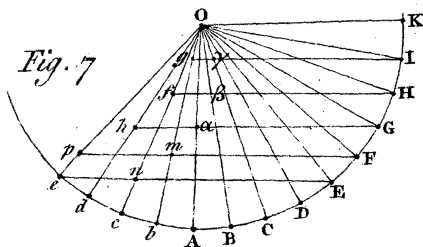
Fig. 2.
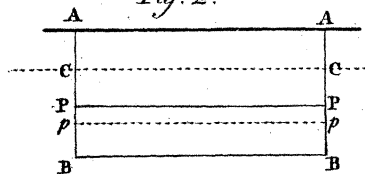
Fig. 3.

Fig. 4.

Fig. 5.

Fig. 6.

Fig. 7

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float-boards, by yielding to the action of the stream, cause the axle on which they are fixed to turn round, by means of several wheels, which take into each other, and give motion to the part destined to produce some purposed effect, as the mill-stone in a corn-mill.

The size of the float-boards, the velocity with which the wheel is to turn, and the number of the float-boards to produce the greatest possible effect, are three main things I propose to examine in the following inquiry.

In the first place I will suppose the total resistance which this wheel has to encounter, on the part of the machine, and which hinders it from moving so swift as the stream, to be expressed by a weight π, suspended to the extremity of a cord fixed to the circumference of a wheel whose radius is d, and which has the same axle as the float-board wheel, so that the effect of the stream is to raise the said weight π, as expressed in TAB. XV. *fig.* 1. I will likewise suppose, that the stream, by its velocity, moves through v feet in one second of time, and that this velocity is the same, though at different depths.

§ II. After these suppositions, the first thing that presents, is to determine what should be the size of the float-boards for the stream to be capable of raising the weight π with a certain determinate velocity.

Let A A B B, *fig.* 2. be one of the float-bords let into the axle A A, and placed vertically in the water, so as to receive the perpendicular impulse of the stream. Its horizontal length $BB = \beta$ feet, its vertical height $AB = \alpha$ feet, the velocity of the

<div align="right">wheel</div>

wheel at the point B, fuch that it fhall run through z feet in a fecond; n pounds the weight of a cubic foot of water; and I will fuppofe the impulfe of the ftream on a plane perpendicular to it is (as Dr. *Daniel Bernoulli* has ftated in his *Hydrodynamica*) equal to the weight of a prifm of water, whofe bafe is the plane, and its altitude the generating height of the velocity with which the plane is impelled. This being fuppofed; let $AP = x$, Pp, its differential, $= dx$, which will give the velocity of the float-board at the point $p = \frac{x}{a}z$, and the relative velocity of the ftream with which the plane is impelled at the fame

point $= v - \frac{x}{a}z$ whofe generating height is $\frac{(v - \frac{x}{a}z)^2}{60}$

feet, whence we have the weight of the parallelopiped of that height, and of the bafe PP pp equal to $\frac{n\beta dx}{60}(-v\frac{x}{a}z)^2$ pounds, which weight multiplied by the length A P (x) of the lever which tends to turn the plane, will give $\frac{n\beta x dx}{60}(v - \frac{x}{a}z)^2$ for the total effect of the ftream on the little rectangle PP pp, whofe integral is $\frac{n\beta}{60}\int (vvx dx - \frac{2vz}{a}xx dx + \frac{zz}{aa}x^3 dx) =$ $\frac{n\beta}{60}\left(\frac{1}{2}vvxx - \frac{2}{3}vz\frac{x^3}{a} + \frac{1}{4}zz\frac{x^4}{aa} - \frac{1}{2}vvff + \frac{2}{3}vz.\right.$ $\left.\frac{f^3}{a} - \frac{1}{4}\frac{zzf^4}{aa}\right)$ (putting A C $= f$ for the diftance between the axle and the furface of the water when the float-board has only its part C B plunged in the water) which (putting $x = a$) will become $\frac{n\beta}{60}\left(\frac{1}{2}vv - \frac{2}{3}vz + \frac{1}{4}zz. aa - \frac{1}{2}vvff + \frac{2}{3}\frac{vzf^3}{aa}\right.$

$- \frac{1}{2}$

$-\frac{1}{4}\frac{zz f^4}{va}$) and will exprefs the effect on the whole plane CC BB, equal π d, the product of the weight π by the length d of the lever on which it acts in oppofing the motion of the wheel.

§ III. If the wheel be plunged as deep as its axle, that is, $f = o$, the equation is changed into this $\frac{n\beta a a}{60}\left(\frac{1}{2}vv - \frac{2}{3}vz + \frac{1}{4}zz\right) = d\pi$, where it appears 1°. That the quantities d, π, v and z remaining the fame, we have β inverfely proportional to the fquare of a, whence it follows, that if the length β is to be diminifhed without altering the effect of the float-board, the height a muft be encreafed proportionally to the fquare root of β; for example, if β is to be made four times lefs, it will be fufficient to double the height a. 2° That likewife the velocity of the float-board remaining the fame, the weight π will be in the compound ratio of the length β, and of the fquare of the height aa. 3° Without meddling with the dimenfions of the float-board, the more the quantity z is increafed, the more muft the weight π be diminifhed. If z be made $= o$, we have $\pi = \frac{n\beta a a}{60d}\cdot\frac{1}{2}vv$, and if $z = v$ we have $\pi = \frac{n\beta a a}{60d}\cdot\frac{1}{12}vv$, that is fix times greater than in the firft cafe; which is very conformable to the nature of things, for when the wheel is in motion, the ftream then not acting upon it but with the excefs of its velocity above that of the wheel, it follows, that the greater fuch velocity is, the more will the effect of the ftream be diminifhed.

It

It follows from our laſt remark, that the greateſt weight with which the ſtream can conſtitute an equilibrium, will be $= \dfrac{n\beta\alpha\alpha\, vv}{120d}$, but then the wheel will not have any motion, nor conſequently the weight π: If the float-board be increaſed, or the weight diminiſhed, from that inſtant the wheel will begin to turn, and the ſwifter as the float-board is greater, or the weight leſs; but in moſt machines, it is required that the weight may be the greateſt poſſible, as alſo the velocity with which it is raiſed. A queſtion therefore here offers itſelf, whoſe ſolution is of much importance. What muſt be the velocity of the float-board whoſe dimenſions are given, that the product of the weight by its velocity ſhall be the greateſt poſſible?

§ IV. The velocity of the weight π is $\dfrac{d}{\alpha}z$ feet in a ſecond, which being multiplied by the value of $\pi = \dfrac{n\beta\alpha\alpha}{60d}\left(\tfrac{1}{2}vv - \tfrac{2}{3}vz + \tfrac{1}{4}zz\right)$ will give the product $\dfrac{n\beta\alpha}{60}\left(\tfrac{1}{2}v^2z - \tfrac{2}{3}vzz + \tfrac{1}{4}z^3\right)$ which muſt be a *maximum*; for which purpoſe make $\tfrac{1}{2}vvdz - \tfrac{4}{3}vzdz + \tfrac{3}{4}z^2\,dz = 0$; whence we have $z = \dfrac{8 - \sqrt{10}}{9} = 0,53752v$: this value of z being ſubſtituted, make the equation $\tfrac{1}{2}vv - \tfrac{2}{3}vz + \tfrac{1}{4}zz = \dfrac{11 + 2\sqrt{10}}{81}vv$, ſo that we have the equation $\beta\alpha\alpha = \dfrac{11 + 2\sqrt{10}}{4860} + \dfrac{d\pi}{nvv} = 280,529 + \dfrac{d\pi}{nvv}$, which expreſſes the dimenſions of the float-board where the effect will be the greateſt poſſible. If the float-board be plunged no deeper than to CC, as we have at firſt ſuppoſed, the moſt advantageous value

of

of z may be determined in the fame manner, which will be found

$$z = \frac{8a^4 - 8af^3 - a\sqrt{10a^6 + 54a^4f^2 - 128a^3f^3 + 54a^2f^4 + 10f^6}}{9(a^4 - f^4)} v$$

If $f = 0$, this value of $z = 0.537v$

$\qquad f = 0,25\, \alpha = \frac{1}{4}\, \alpha. \qquad = 0,498v$

$\qquad f = 0,3\, \alpha \qquad\qquad = 0,486v$

$\qquad f = 0,5\, \alpha = \frac{1}{2}\, \alpha. \qquad = 0,436v$

$\qquad f = 0,7\, \alpha \qquad\qquad = 0,390v$

$\qquad f = 0,9\, \alpha \qquad\qquad = 0,353v$

$\qquad f = \alpha \qquad\qquad\qquad = 0,333v$

By the infpection of thefe different values it appears that this value of z diminifhes as the plunged part is greater, and that this velocity can never exceed the quantity $0,537v$, nor be lefs than $\frac{1}{3}v$ *.

This value of z and of its fquare zz being fubftituted in the general formula (§ II.) we fhall obtain from it the following equation:

$$\frac{60d\pi}{\alpha\beta vv} = \frac{11a^6 - 27a^4f^2 + 32af^3 - 27a^2f^4 + 11f^6 + 2(a^3 - aff)\sqrt{10a^6 + 54a^4f^2 - 128a^3f^3 + 54a^2f^4 + 10f^6}}{81(a^4 - f^4)}$$

which for a given relation between f and α will fhew the breadth β for producing the greateft effect.

* If in the value of z we make $f = \alpha$, we have $z = \frac{0}{0}$ which obliges us to take, according to the common method, the differentials of the numerator and of the denominator, confidering f as variable, and the relation of thefe differentials will give the value of z: but on account of the radical quantity, the calculus being fomewhat tedious, and, again bringing out $z = \frac{0}{0}$ and that after feveral fimilar operations, it is better to have recourfe to the equation from which the value of z was deduced; this equation is $\frac{3}{4}\,\alpha v z.\frac{\alpha^3 - f^3}{\alpha^4 - f^4} - \frac{1}{2}\,\alpha\alpha vv.\frac{\alpha^2 - f^2}{\alpha^4 + f^4}$, which by the above operation will be $\frac{3}{4}zz = vz - \frac{1}{4}vv$, and $z = \frac{1}{3}v$.

As the extremity of the float-board muſt have a certain velocity depending on the relation of the height a to the plunged part, and as the velocity of the weight $\pi = \dfrac{d}{a} z$, it follows that if we would increaſe the velocity of π, we muſt diminiſh the height a and increaſe the breadth β, ſo that the product $\beta a a$ and the relation of f to a may be the ſame as before ; for example, if the wheel be plunged as deep as the axle, to double the velocity of the weight, the height of the float-board muſt be reduced one half, and its breadth be quadrupled.

§ V. It may ſo happen that the channel on which the wheel is placed ſhall be ſo ſhallow and narrow, as not to allow the float-boards the neceſſary dimenſions, for raiſing the weight with a convenient velocity. In this caſe we are obliged to raiſe the axle of the wheel above the ſurface of the water, ſo much that the lever on which the ſtream acts may be long enough to recompenſe the ſmallneſs of the float-boards. Herein it is neceſſary to ſolve the following problem :

The breadth b, *and the height* a, *of the float-board* A B *being given*; *to find the radius* C A (r) *of the wheel which ſhall cauſe the weight* π *to aſcend with the velocity* $\dfrac{d}{r + a} z$.

The exact ſolution of this problem might be deduced from the formula (§ II.) which would render the operation tedious, the equation being of the fourth degree ; but it may be rendered far more ſimple by a ſuppoſition which is but little wide of the truth when A B is but ſmall in compariſon of C A, and this is to

conſider

confider all the points of the float-board A B as affected with the fame velocity z.

Let C P fig. 3. $= x$, we fhall have $\frac{nb}{60} (v-z)^2 \int x\, dx$ for the effect of the portion AP, and $\frac{nb}{60} (v-z)^2$

$\times \frac{2ar+aa}{2}$ for the effect of the whole float-board AB. This quantity muft be made equal to d π, and then, juft as in the foregoing cafes, fuch a value of z be fought, that the weight π and its velocity may be the greateft poffible ; that is, the differential of $z (v-z)^2$ muft be made $= 0$, which gives $z = \frac{1}{3} v$. Therefore d $\pi = \frac{nabvv}{270} (2r+a)$, and $r = \frac{270\,\mathrm{d}\pi - naabvv}{2nabvv}$, and the velocity of the weight π will be $=$ $\frac{2nd abvv}{810\,\mathrm{d}\pi + 3nbaav} \times v$.

§ VI. We have feen that the calculus was much fimplified by fuppofing one of the velocities conftant for all points of the float-board. For this velocity being c, the effect of the whole float-board will be fimply $\frac{n\beta}{60} (vc)^2 \frac{\alpha\alpha - ff}{2}$. It will therefore not be unufeful to inquire what this velocity c muft be, that the effect of the float-board may be the fame, as fuppofing, as we have hitherto done, a variable velocity, and proportional to the diftances from the axle, we have only to make $\frac{n\beta}{60} \overline{(v-c)}^2 \left(\frac{\alpha\alpha - ff}{2} \right)$

$= \frac{n\beta}{60} \left(\frac{1}{2} vv (\alpha\alpha - ff) - \frac{2}{3} vz \frac{a^3 - f^3}{\alpha} + \frac{1}{4} zz \frac{\alpha^4 - f^4}{\alpha\alpha} \right)$

(§ IV.) but the equation whence we got the value of z

($\S$ IV.) $\frac{1}{2} vv - \frac{4}{3} \frac{vz}{a} \cdot \frac{a^3 - f^3}{a^2 - f^2} + \frac{3}{4} zz \cdot \frac{aa + ff}{aa} = 0$, we

shall have $c = v - \sqrt{\frac{1}{2} vv - \frac{1}{4} zz \cdot \frac{aa + ff}{aa}}$.

If $f = 0$, we have $zz = 0,288 vv$ and $c = 0,345 v$

$\quad f = \frac{1}{2} a \qquad\qquad zz = 0,190 vv \qquad\quad c = 0,336 v$

$\quad f = a \qquad\qquad zz = \frac{1}{9} vv \qquad\qquad c = \frac{1}{3} v.$

fo that whatever be the relation of f to a, the velocity c is ever nearly $= \frac{1}{3} v$, and the more exactly fo, as f is greater. Wherefore we may always affume $\frac{1}{210} n\beta vv. (aa - ff) = d\pi$ for the effect of the ftream upon a float-board whofe plunged part is $a - f$; this effect will be increafed in the ratio of 4 to 9, when the wheel has no motion, for making $c = 0$, we find it $= \frac{1}{120}. n\beta vv. (aa - ff)$.

$\S$ VII. Hitherto we have all along fuppofed, that the float-board did through its whole plunged part receive the perpendicular impulfe of the ftream; but it is eafily underftood, that the wheel coming to turn, prefents to the ftream the plane of the float-board under an angle which is continually varying, which diminifhes its effect every inftant, as it removes from the vertical: This inconvenience may be remedied by multiplying the number of the float-boards, fo that when the firft is removed from the vertical as far as a certain point, the next may occupy that advantageous place, to be in its turn replaced fome time after by a third, and fo on. Now our third inquiry is, to affign the angle contained between two float-boards, or, which comes to the fame, the number of float-boards the wheel fhould confift of, that its effect may be the greateft poffible, being of no lefs importance than the preceding ones. To begin then with the moft

fimple

fimple cafe; we will fuppofe the wheel immoveable, or that $c = 0$, and proceed to inveftigate, whether, fuppofing the number of float-boards to be greater, the fum of the effects will come out greater or lefs than what refults from one fingle float-board placed vertically.

In order to a general folution of this queftion, we will fuppofe two float-boards CD and CE fig. 4. making any angles with the vertical, and let us compare the effect of the fingle float-board GD with the effect refulting from the float-boards FE and GD taken together, which will be reduced to FE and OD, becaufe the part OG becomes ufelefs, as the ftream is intercepted by FE. Let $CB = CD = CE = a$, $CA = f$, cofin. $BCD = m$, cofin. $BCE = \mu$, which gives $CG = \frac{f}{m}$ $CF = \frac{f}{\mu}$ and $CO = \frac{\mu}{m} a$. Then we fhall find, by § VI, the effect of $GD = \frac{n\beta vv}{120} (mm\,\alpha\alpha - ff)$ that of $OD = \frac{n\beta vv}{120} (mm\,\alpha\alpha - \mu\mu\,\alpha\alpha)$. and that of $FE = \frac{n\beta vv}{120} (\mu\mu\,\alpha\alpha - ff)$; whence it appears, that the fum of the two laft is exactly equal to the firft, which will ever hold good whatever be the value of f.

Whence arifes the following theorem:

Whether the wheel be plunged quite up to the axle, or only in part fo, provided it be immoveable, and that one of its float-boards be placed vertically, its effect will be conftantly the fame, whatever be the number of float-boards oppofed to the ftream, even though it were infinite.

The

The latter part of this theorem, though flowing from the general demonftration, may be alfo demonftrated, immediately, in the following manner; Let BP fig. 5. be $= x$, we have $MO = \frac{adx}{a-x}$, and $CO = \frac{aa - dx - ax}{a - x}$;

$\overline{CO}^2 = \frac{a^4 - 2a^3x + a^2x^2 - 2a^3dx + 2aaxdx}{aa - 2ax + xx}$ neglecting the dx^2) and $aa - \overline{CO}^2 = \frac{2aaxdx}{a-v}$; Therefore the effect of the ftream upon OM, which is $= \frac{n\beta vv}{120} \cdot (aa - \overline{CO}^2) \frac{\overline{CP}^2}{\overline{CP}^2}$ will become $= \frac{n\beta vv}{120} (2ax - 2xdx)$ whofe integral is $= \frac{n\beta vv}{120}(2ax - xx)$ wherein puttting $x = a - f$, we have $\frac{n\beta vv}{120}(aa - ff)$ for the total effect of the ftream upon the wheel, which is the fame as that of a fingle float-board A B in a vertical pofition.

§ VIII. This theorem will alfo hold true for the cafe of § V. wherein we have fuppofed the height of the float-boards very fmall, in comparifon of the radius of the wheel; we have feen that the effect of a fingle float-board placed vertically was $= nabvv$ $(2r + a)$; the demonftration of the preceding § will be applicable here after the fame manner, and will fhew that whatever be the number of float-boards, the effect will be ever the fame.

It does not however follow that the number of float-boards fhould be indifferent; for the wheel coming to turn the float-board, its lower part, which received the perpendicular impulfe, will no longer

receive

receive it otherwife than obliquely, and the effect will diminifh till the angle formed by two neighbouring float-boards be bifected exactly by the vertical, which will render the firft entirely ufelefs; after which the effect will increafe anew, and will become again greateft, when the fecond float-board is got to the vertical; fo that in order to fix upon the moft advantageous number of float-boards, regard muft be had to the fum of the different effects for all the fituations of the float-boards during one whole turn of the wheel.

Whence it follows that in this cafe, wherein they are fuppofed very fmall, the greater their number is, the greater will be the fum total of the effects, fince, if that number were infinite, there would be a float-board in a vertical pofition every inftant.

§ IX. This will no longer hold good, if the height of the float-boards be more confiderable, and it be found neceffary to take the different velocity of their different points into confideration; by comparing (fig. 4.) the preffure on F E with that on the portion G O, they will be found no longer equal, as in the foregoing cafe; it is true that the fame quantity of fluid acts on thefe two planes, and the difadvantage which F E has by receiving the impulfe more obliquely, is exactly compenfated, as before, by the length of the lever, but the difference arifes from the different velocity of the correfponding points of F E and G O; thofe velocities are in the ratio of C F to C G, or as cof. A C G to cof. A C F, which fhews that the effect of F E is always lefs than that of G O, and confequently the effect muft be diminifhed, by adding a greater number of float-boards;
the

the faid effect will be greateſt when there is only one float-board placed vertically, and leaſt when their number is infinite : let us enquire what it will be in this latter caſe. We will ſuppoſe the ſame fig. 5, and the ſame denominations as in § VII. We had $CO = \frac{aa - ax - adx}{a - x}$, we ſhall have (neglect-

ing dx^2, dx^3, and dx^4) $aa - \overline{CO}^2 = \frac{2aadx}{a-x}$,

$\frac{a^3 - \overline{CO}^3}{a}$, and $\frac{a^4 - \overline{CO}^4}{aa} = \frac{4aadx}{a-x}$. Now the preſ-

ſure on OM is, by § II, $= \frac{n\beta}{120} \overline{\frac{a-x}{aa}}^2 \Big(vv \, (\, aa$

$- \overline{CO}^2 \big) - \frac{4}{3} vz. \frac{a^3 - \overline{CO}^3}{a} + \frac{1}{2} zz. a^4 - \frac{\overline{CO}^4}{aa} \Big)$

which (by putting for CO its value) will become

$= \frac{n\beta}{120} (2adx - 2xdx) \, (v - z^2)$ whoſe integral

$\frac{n\beta}{120} (2ax - xx) \, (v - z^2) = \frac{n\beta \, (v - z^2)}{120} (aa - ff)$

(making $x = a - f$) will expreſs the effect re-ſulting from an infinite number of float-boards : this leaſt effect will be to the greateſt, that is when there is but one float-board, as $(v - z^2) : vv -$

$\frac{4}{3} \frac{vz}{a} . \frac{a^3 - f^3}{aa - ff} + \frac{1}{2} \frac{zz}{aa} . (aa + ff)$ or as $(v - x^2)$

$: \frac{1}{2} vv - \frac{1}{4} zz. \Big(\frac{aa + ff}{aa} \Big)$ §. VI.

This ratio will be that of $1 : 2$ if $f = 0$

$\qquad\qquad\qquad\qquad\qquad 1 : 1,485 \qquad f = \frac{1}{2}a$

$\qquad\qquad\qquad\qquad\qquad 1 : 1 \qquad\quad f = a$ If

§ X.

§ X. If we take nothing but the moft advantageous pofition into confideration, and preferve the greateft effect entire, it follows that the angle BCD (fig. 6.) between two float-boards muft be fuch that E fhould enter the water at the inftant when A B quits the vertical, fo that the cofine of that angle be

$= \frac{f}{\alpha}$; in confequence of which the following table

may be conftructed, fhewing what the number of float-boards fhould be for a given ratio between f and α.

For 4 float-boards, we have $f =$ 0,

5	0,3090α
6	0,5000α
7	0,6236α
8	0,7071α
9	0,7660α
10	0,8090α
12	0,8669α
14	0,9009α
16	0,9239α
18	0,9397α
20	0,9510α
&c.	&c.

§ XI. Certain authors treating of hydraulics, have in this part thereof given the fame table, as containing the true number of float-boards the wheel fhould confift of: but we have feen upon what principle it was formed, and that it was only to preferve entirely the effect of the vertical float-board; from whence it follows not that the number of float-boards which it affigns fhould be the moft advanta-

geous. To which purpofe the effect produced from every pofition of the wheel, and for the different number of the float-boards, fhould be computed; the number which gives the arithmetical mean between all thefe effects, the greateft of all, will be that to be chofen, and preferred before what the above table indicates.

It may be fufficiently fatisfactory to compute only the effect from 1 to 10 degrees. Thus, for example, for the wheel entirely plunged, we are to find the effect (fig. 7.)

1° on O A, 2° on O I and gb, 3° on O H, and fc, 4° on O G and bd, 5° on O F, and pe, 6° on O E, 7° on O D, 8° on O C, and 9° on O B.

After which the wheel returns into the fame pofition it had at firft; and we are to divide the fum of all thefe effects by 9, to get the arithmetical mean.

We will next fuppofe the number of fix float-boards for the fame cafe of $f = o$, and compute the following effects.

1° on O G $+ a$ A, 3° on O E $+ nc$, 5° on O I $+ \gamma$ C
2°   O F $+ mb$, 4°   O D   6°   O H $+ \beta$ B.

The fum of all thefe effects divided by 6 will give the effect of the wheel of 6 float-boards.

The fame thing, fuppofing the angle 40 degrees, or 9 float-boards, and as after a revolution of thefe 40 degrees, the wheel returns into a fimilar pofition, the fame muft be divided by 4.

Then for an angle of 30 degrees we are to divide by 3, and fo on.

for

I have made this computation to great exactnefs, for the cafe of $f = 0$, $f = \frac{1}{4}\alpha$, and $f = 0$, $866\,\alpha = \alpha$ cof. $30°$; the refult,

1° If $f = 0$.

for 4 float-boards, the arith. mean $= 0.335$
$(\frac{1}{40}\ n\beta\alpha\alpha vv)$.

It may be obferved in this firft cafe, that there is fome advantage in taking 6 float-boards inftead of 4 fhewn by the table; the effect will be increafed in the ratio of 100 to 118, and yet will be more than about $\frac{9}{100}$ of the greateft effect above calculated for a fingle vertical float-board; fo that the found dimenfions muft be a fmall matter altered, and the quantity $\beta\alpha\alpha$ increafed by $\frac{1}{10}$.

for 6	$= 0,396\ (\&c.$
9	$= 0,336\ (\&c.$
12	$= 0,323\ (\&c.$
18	$= 0,295\ (\&c.$
an infinite number	$= 0,214\ (\&c.$

2° If $f = \frac{1}{4}\alpha$.

for 6 flo.	$0,277\ (\&c.$
7	$0,281\ (\&c.$
9	$0,285\ (\&c.$
12	$0,284\ (\&c.$
18	$0,276\ (\&c.$
an infinite number	$0,238\ (\&c.$

In this fecond cafe 9 float-boards are to be taken inftead of 6 fhewn by the table, though the difference will be but very fmall; and we fhall have an effect which will be $\frac{86}{100}$ of that of a vertical float-board, and in that ratio that the quantity $\beta\alpha\alpha$ found by the above formulæ, muft be increafed.

3° If

3° If $f = 0,866\ a$

for 12 fl. $= 0,099\ (\&c.$

18 $= 0,099\ (\&c.$

36 $= 0,104\ (\&c.$

an infinite number $= 0,103\ (\&c.$

In this third cafe the difference is ftill very fmall, and the effect refulting from 36 float-boards will be $\frac{0,1}{10,0}$ of the effect of a fingle vertical float-board.

XXXVI.